Mathematics 101

Solved Past Questions with Calculator Method

Vickies Foundation

August 2020

MATHEMATICS 101

This document is prepared by one of the University Representatives of Vickies Foundation, a Student of Ladoke Akintola University. He took the initiative to help his colleagues and every other student in all parts of Nigeria with this document which is an easier way to solve mathematical problems. This document is relevant for University students.

Vickies Foundation made this document available during the COVID-19 Pandemic to help students prepare ahead of school re-opening and examinations.

Published By:

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FEATURES ON THE CALCULATOR AND THEIR FUNCTIONS

Mode Initialization

The calculator mode can be returned to the initial default pressing the

[Shift] ⟶ [CLR] ⟶ [2] ⟶ [=] ⟶ [AC]

Store and Recall

To store a number or answer on the calculator, the following procedure must be followed.

Click on the number you want to store and press [=]

Then press [Shift] ⟶ [STO] ⟶ [The Alphabet you want to save your number with e.g. A, B, X, Y]

E.g. [2] ⟶ [=] ⟶ [Shift] ⟶ [STO] ⟶ [A] = (A = 2)

MODE OF EACH CALCULATIONS

MODE Description	KEY Operation	Notation
Basic Arithmetic	[MODE] ⟶ [1]	COMP
Complex Number	[MODE] ⟶ [2]	CMPLX
Statistical Calculation	[MODE] ⟶ [3]	STAT
Base N. Calculation	[MODE] ⟶ [4]	BASE
Equation	[MODE] ⟶ [5]	EQN
Matrix Calculation	[MODE] ⟶ [6]	MAT
Table Calculation	[MODE] ⟶ [7]	TABLE
Vector Calculation	[MODE] ⟶ [8]	VCT
Decimal Place	[Shift] ⟶ [MODE] ⟶ [6]	Fix
Significant Figure	[Shift] ⟶ [MODE] ⟶ [7]	SCi

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CHAPTER ONE

THEORY OF QUADRATIC

$$[ax^2 + bx + c]$$

Example 1: Given that $ax^2 + bx - c = 0$. Find;

(a) $\alpha + \beta$

(b) $\alpha\beta$

Solution:

(a) $\alpha + \beta = \frac{-b}{a}$

(b) $\alpha\beta = \frac{c}{a}$

Example 2: If $\alpha \; and \; \beta$ are the root of the equation $4x^2 + 2x - 3 = 0$. Find;

(a) $\alpha + \beta$

(b) $\alpha\beta$

(c) $\frac{\alpha^3}{\beta^3} + \frac{\beta^3}{\alpha^3}$

Solution:

- On your calculator, press [MODE] ⟶ 5 (equation) ⟶ 3
- Then input the values for a, b, c
  which is a = 4, b = 2, c = −3

$$x_1 = \frac{1 + \sqrt{13}}{4}, x_2 = \frac{-1 - \sqrt{13}}{4}$$

- Then go back to the main screen by pressing [AC]

Note: Do not press [On] because it will reset the calculator.

$$\therefore \quad x_1 = \alpha, x_2 = \beta$$

- We can also save our $x_1 \; as \; \alpha$ and $x_2 \; as \; \beta$ to make our work faster.

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Note: We do not have the α and β key on the calculator, so we are going to substitute α and β for A and B

- Now, Press $\frac{-1+\sqrt{13}}{4} \longrightarrow$ [Shift] $\longrightarrow$ [Rcl] $\longrightarrow$ [A] $\left[A = \frac{-1+\sqrt{13}}{4}\right]$

  $\frac{-1+\sqrt{13}}{4} \longrightarrow$ [Shift] $\longrightarrow$ [Rcl] $\longrightarrow$ [B] $\left[B = \frac{-1+\sqrt{13}}{4}\right]$ then [AC]

Question 1:

Find $\alpha + \beta \longleftrightarrow$ Find A + B

Since your answer has been saved as A and B

Press A + B on your calculator

[Alpha] $\longrightarrow$ [A] + [Alpha] $\longrightarrow$ [B] = $\mathbf{\frac{-1}{2}}$

Question 2:

Find AB

Press [Alpha] $\longrightarrow$ [A] × [Alpha] $\longrightarrow$ [B] = $\mathbf{\frac{-3}{4}}$

Question 3:

Find $\frac{A^3}{B^3} + \frac{B^3}{A^3}$

Press the division key [÷] then insert $\frac{A^3}{B^3} + \frac{B^3}{A^3}$ = $\mathbf{\frac{-154}{27}}$

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2ND METHOD

[This only works for $\alpha + \beta$ and $\alpha\beta$]

Question 1:

Find $\alpha + \beta$ from the equation $4x^2 + 2x - 3 = 0$

From (Example 1) we said $\alpha + \beta = \frac{-b}{a}$, while $\alpha\beta = \frac{c}{a}$

Therefore; $4x^2 + 2x - 3 = 0$

$$\alpha + \beta = \frac{-b}{a} = \frac{-2}{4} = \mathbf{\frac{-1}{2}}$$

$$\alpha\beta = \frac{c}{a} = \mathbf{\frac{-3}{4}}$$

Example 3: Given that $6x^2 - x - 2 = 0$. Find;

(a) $\alpha + \beta$

(b) $\alpha\beta$

Solution:

From (Example 1) $\alpha + \beta = \frac{-b}{a}$, while $\alpha\beta = \frac{c}{a}$

$$\alpha + \beta = \frac{-b}{a} = -\left[\frac{-1}{6}\right] = \mathbf{\frac{1}{6}}$$

$$\alpha\beta = \frac{c}{a} = \frac{-2}{6} = \mathbf{\frac{-1}{3}}$$

Example 4: If $\alpha + \beta = \frac{1}{6}$ and $\alpha\beta = \frac{-1}{3}$, find the quadratic equation of the root $\alpha + \beta$ and $\alpha\beta$

Formula:

x^2 – [sum of the roots]x + [product of the roots] = 0

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Therefore, sum of the root $= \alpha + \beta = \frac{1}{6}$

product of the root $= \alpha\beta = \frac{-1}{3}$

Insert the values into the equation

x^2 – [sum of the roots]x + [product of the roots] $= 0$

$x^2 - \left[\frac{1}{6}\right]x + \left[\frac{-1}{3}\right] = 0$

$x^2 - \frac{1}{6x} - \frac{1}{3} = 0$

Multiply all by the highest denominator which is 6

$6 \times \left[x^2 - \frac{1}{6x} - \frac{1}{3}\right] = 0$

$6x^2 - \frac{\cancel{6}}{\cancel{6}x} - \frac{\cancel{6}}{\cancel{3}} = 0$

$\mathbf{6x^2 - x - 2 = 0}$

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CHAPTER TWO

OPERATIONS OF REAL NUMBERS (SURD)

Example 1; Find the square root of $5 + 2\sqrt{6}$

$x^2 -$ [Sum of the root] $x +$ Product of the root $= 0$

$5 + 2\sqrt{6}$

$x^2 = 2$

Now we look for the sum and product of the root

$3 + 2 = 5$

$3 \times 2 = 6$

$\therefore \sqrt{3} + \sqrt{2}$ is the square root of $5 + 2\sqrt{6}$

OR

Find the square root of $5 + 2\sqrt{6}$

Press your [$\sqrt{\ }$] square root key and insert your values

$\sqrt{5 + 2\sqrt{6}} = 3.1462$

Test the options $\sqrt{3} + \sqrt{2} = 3.1462$

Example 2; Find the square root of $28 + 2\sqrt{195}$

$x^2 = 2$

Sum of the root

$15 + 13 = 28$

Product of the root 15 and 13

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$15 \times 13 = 195$

$\therefore \sqrt{15} + \sqrt{13}$ is the square root of $28 + 2\sqrt{195}$

OR

$\sqrt{28 + 2\sqrt{195}} = 7.4785$

Test the options $\sqrt{\mathbf{15}} + \sqrt{\mathbf{13}} = \mathbf{7.4785}$

Example 3; Rationalization

Find the value of $\frac{1}{3-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-2}$

(a) 2

(b) 3

(c) 4

(d) 5

Press all the equation on the calculator and press [=], the answer is **5**.

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CHAPTER THREE

INDICES AND LOGARITHM

Example 1: Find the value of x in the equation of $9^{1+x} \times 3 = 27^{-x}$

(a) $^{3}/_{5}$

(b) $^{-3}/_{5}$

(c) $^{-4}/_{6}$

(d) $^{4}/_{6}$

Note: The [=] sign in indices and logarithm is different. To get this [=] we are going to use [Alpha] ⟶ [Calc.]

N.B: There are two ways of solving questions like this, which will be shown below.

METHOD 1

- Press the whole equation on your calculator.
- Which is $9^{1+x} \times 3$ ⟶ [Alpha] ⟶ [Calc] ⟶ 27^{-x}

  This is also the same as $9^{1+x} \times 3 = 27^{-x}$
- Since we are solving for x, we would press [Shift] ⟶ [Calc]

  Which is; solve for x, then [=]
- The answer here is -0.6
- Then start testing your options.

  Option A $^{3}/_{5} = 0.6$

  Option B $^{-3}/_{5} = -0.6$

  The Answer is O**ption B.**

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METHOD 2

Note: This method is a little bit confusing but easier.

Find the value of x in the equation of $9^{1+x} \times 3 = 27^{-x}$

N.B: Using this method, we will not be solving for x though, but we are going to test the options by inserting them into x.

$9^{1+x} \times 3 = 27^{-x}$

Equate this to zero

Main Question $9^{1+x} \times 3 - 27^{-x} = 0$

Then press the equation above on your calculator excluding the $=0$

- $9^{1+x} \times 3 - 27^{-x}$, then press [Calc]
- Your calculator would pop up [X?]. Do not mind the numbers at the bottom right corner, they are irrelevant.
- **N.B**: Our answer must be equal to zero as seen above in the main question
- This is where we start inserting our options

(a) X? $\left[{}^{3}/_{5}\right] = 100.765$

(b) X? $\left[{}^{-3}/_{5}\right] = 0$

(c) X? $\left[{}^{-4}/_{6}\right] = -2.759$

Note: ${}^{-3}/_{5} = 0$

So, our answer is $\mathbf{{}^{-3}/_{5}}$

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Example 2: Solve for n in $4^{n-1} \times 5^{2n-2} \times 10^{n} = 1$. Find n

- Another easy way of doing this.

(a) $^{2}/_{3}$

(b) $^{3}/_{2}$

(c) $^{4}/_{2}$

N.B: We do not have "n" on our calculator, so we substitute n for x.

Hence; $4^{x-1} \times 5^{2x-2} \times 10^{x} - 1 = 0$

- Insert the options. Any option you insert must give you zero.

Option A $[^{2}/_{3}] = 4^{^{2}/_{3}-1} \times 5^{2[^{2}/_{3}-2]} \times 10^{x} - 1 = 0$

Option B $[^{3}/_{2}] = 4^{^{3}/_{2}-1} \times 5^{2[^{3}/_{2}-2]} \times 10^{^{3}/_{2}} - 1 = 3.15$

Option C $[^{4}/_{2}] = 4^{^{4}/_{2}-1} \times 5^{2[^{4}/_{2}-2]} \times 10^{^{4}/_{2}} - 1 = 9.9$

Therefore; $^{2}/_{3} = 0$

Answer = $\mathbf{^{2}/_{3}}$

Example 3: $5\ log_{a}^{y} - 2\ log_{a}(x+4) = 2log_{a}^{y} + log_{a}^{x}$

(a) $\sqrt[3]{x(x+4)^{2}}$

(b) $\sqrt[3]{(x+4)^{2}}$

(c) $\sqrt[3]{x(x+4)}$

(d) $\sqrt[3]{x+4}$

∴ We know any Logarithm without a Base is automatically in Base 10

$a = 10$

Our biggest number in the equation would be x

$x = 5$

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So, we are left with Y = ?

We want to solve for Y

But then we do not have Y on our calculator. So anywhere we see Y we substitute it for x.

The equation would be like this

$5\ log_{10}^{x} - 2\ log_{10}(5+4) = \ 2log_{10}^{x} + log_{10}^{5}$

Press all these on your calculator, then solve for x which is

[Alpha] ⟶ [Calc]

The answer is **7.3986**

You are surprised? Don't be. Let us test our options since we have the value of x.

Option A $\sqrt[3]{5(5+4)^2} = 7.3986$ ✓

Option B $\sqrt[3]{(5+4)^2} = 4.3267$ ✗

Option C $\sqrt[3]{5(5+4)} = 3.5568$ ✗

Therefore, our answer is Option A $\sqrt[3]{\boldsymbol{x(x+4)^2}}$

Example 4: If $2log_4^p + \ log_4^{q^2} = 0$. Find P = ?

(a) $^1/_P$

(b) $^1/_{log\ q}$

(c) $^1/_{log\ P}$

(d) $^1/_q$

N.B: Biggest number in the equation would be q

q = 2

Substitute p for x in the equation

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$2log_4^x + log_4^{2^2} = 0$

$2log_4^x + log_4^{2^2}$ ⟶ [Alpha] ⟶ [Calc] ⟶ 0

Then, press solve for x which is [Shift] ⟶ [Calc]

The answer is **0.5**

Let us test our options since we have q = 2

Option A $^1/_P$ ∝ (We do not have the value for p)

Option B $^1/_{log\ 2} = 3.321$ ∝

Option C $^1/_{log\ P} =$ ∝

Option D $^1/_2 = 0.5$ ✓

∴ $^1/_q$ is our final answer

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CHAPTER FOUR

PARTIAL FRACTION

We have three different types of partial fraction

1. Distinct Linear Factors
2. Repeated Linear Factors
3. Irreducible Factors

Example 1: Distinct Linear Factors

$$\frac{x+7}{x^2-7x+10} = \frac{\mathbf{4}}{\boldsymbol{x-5}} - \frac{\mathbf{3}}{\boldsymbol{x-2}} \longrightarrow Ans.$$

Solution

Substitute zero (0) for x

i.e anywhere you see x insert zero (0)

$$\frac{0+7}{0^2-7(0)+10} = \frac{7}{10}$$

Pick the answers too and insert zero (0) for x

$$\frac{4}{0-5} - \frac{3}{0-2} = \frac{\mathbf{7}}{\mathbf{10}}$$

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Example 2: Irreducible Factors

$$\frac{5x^2+9x-1}{(2x+3)(x^2+5x+2)}=\frac{1}{2x+3}+\frac{2x-1}{x^2+5x+2}$$

Solution

Substitute 0 for x

$$\frac{5(0)^2+9(0)-1}{(2(0)+3)(0^2+5(0)+2)}=\frac{-1}{6}$$

Answer

$$\frac{1}{2(0)+3}+\frac{2(0)-1}{0^2+5(0)+2}=\frac{\mathbf{-1}}{\mathbf{6}}$$

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CHAPTER FIVE

MATRICES

Curly Bracket

$$\begin{bmatrix} 4 & 1 & 2 \\ 6 & 7 & 8 \\ 1 & 4 & 6 \end{bmatrix} = 3 \times 3 \text{ Matrix}$$

$$[1 \quad 4 \quad 6] = 1 \times 3 \text{ Matrix}$$

$$\begin{bmatrix} 6 \\ 2 \\ 8 \end{bmatrix} = 3 \times 1 \text{ Matrix}$$

Square Matrix are;

$2 \times 2, 3 \times 3, 4 \times 4 \ldots$

Singular Matrix are

the Matrix which determinant is equal to zero (0).

Symmetric Matrix:

This is a type of Matrix which is equal to its transpose.

Example of Symmetric Matrix

$$\text{A} = \begin{vmatrix} 0 & 1 & 4 \\ 1 & 0 & 2 \\ 4 & 2 & 1 \end{vmatrix}$$

$$A^T = \begin{vmatrix} 0 & 1 & 4 \\ 1 & 0 & 2 \\ 4 & 2 & 1 \end{vmatrix}$$

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Skilled Symmetric

A Matrix is said to be skilled if it is equal to the transpose in the opposite sign.

$$B = \begin{vmatrix} 0 & -1 & 4 \\ 1 & 0 & -6 \\ -4 & 6 & 0 \end{vmatrix}$$

$$B^T = \begin{vmatrix} 0 & -1 & 4 \\ 1 & 0 & -6 \\ -4 & 6 & 0 \end{vmatrix}$$

Determinant of the Matrix

The determinant of the Matrix is usually represented with this sign $|A|$

$|A|$ = Determinant of Matrix A

Example 1: Find the determinant of Matrix A

$$\text{If } A = \begin{vmatrix} 4 & 5 & -2 \\ 6 & -1 & 5 \\ 2 & 3 & 7 \end{vmatrix}$$

This question will be solved in two ways.

i. Manually
ii. With calculator

Manually

$$A = \begin{vmatrix} (+) & (-) & (+) \\ 4 & 5 & -2 \\ 6 & -1 & 5 \\ 2 & 3 & 7 \end{vmatrix}$$

In a determinant of Matrix, it is important to put the (+) (-) (+) sign on each row as shown above.

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$$\begin{vmatrix} (+) & (-) & (+) \\ 4 & 5 & -2 \\ 6 & -1 & 5 \\ 2 & 3 & 7 \end{vmatrix} \qquad \begin{vmatrix} (+) & (-) & (+) \\ 4 & 5 & -2 \\ 6 & -1 & 5 \\ 2 & 3 & 7 \end{vmatrix} \qquad \begin{vmatrix} (+) & (-) & (+) \\ 4 & 5 & -2 \\ 6 & -1 & 5 \\ 2 & 3 & 7 \end{vmatrix}$$

$= +4[-1 \times 7 - 3 \times 5] - 5[6 \times 7 - 2 \times 5] - 2[6 \times 3 - 2 \times (-1)]$

$= -88 - 160 - 40$

$= \mathbf{-288}$

<u>With Calculator</u>

- Press [MODE] ⟶ [6, MATRIX] ⟶ [1, MAT A] ⟶ [1, 3 × 3]

Then insert the values.

$$A = \begin{vmatrix} 4 & 5 & -2 \\ 6 & -1 & 5 \\ 2 & 3 & 7 \end{vmatrix}$$

- After inserting this, press [=]
- Then press AC

N:B: If you press the [ON] button, it will delete the saved equation

- Now Press [Shift] ⟶ [4] ⟶ [7] ⟶ [Shift] ⟶ [4] ⟶ [3]
- Then press [=]

The Answer is $\mathbf{-288}$.

N:B: We chose MAT A because we solved our equation on MAT A.

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Example 2: Addition of Matrix

If $A = \begin{vmatrix} 1 & 4 \\ 3 & 2 \end{vmatrix}$

$B = \begin{vmatrix} 4 & 2 \\ 1 & 5 \end{vmatrix}$

Find A + B

Solution

Manually

$$\begin{vmatrix} 1 & 4 \\ 3 & 2 \end{vmatrix} + \begin{vmatrix} 4 & 2 \\ 1 & 5 \end{vmatrix} = \begin{vmatrix} \mathbf{5} & \mathbf{6} \\ \mathbf{4} & \mathbf{7} \end{vmatrix}$$

With Calculator

- Press [MODE] ⟶ [6] ⟶ [1] ⟶ [5, 2 × 2], then insert the values
- After that, Press [=] ⟶ [AC]
- Then press this: [MODE] ⟶ [6] ⟶ [2, MAT B] ⟶ [5, 2 × 2], then insert the values and press [=] ⟶ [AC]
- Press [Shift] ⟶ [4] ⟶ [3] ⟶ [Shift] ⟶ [4] ⟶ [4] ⟶ [=]

The answer is $\begin{vmatrix} \mathbf{5} & \mathbf{6} \\ \mathbf{4} & \mathbf{7} \end{vmatrix}$

N:B: The calculator might show dimension error. If this happens, what you must do is

Press [Shift] ⟶ [4] ⟶ [1] and insert the values again.

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